Multinational Production, Risk Sharing, and Home Equity Bias

Technical Appendix

Not for Publication

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1 Model Details

This Appendix shows derivations for Section 2.

1.1 Derivation of price indices, demand for goods, and real exchange rate

First, we derive the price index in the home country, P_t. It consists of the price index of goods produced by home rms in the home country, P_{Ht}, and price index of goods produced by foreign rms in the home country, P_{Ft}. C_t is the home consumer's consumption basket

Now, we derive the price index of goods produced by home firms in the home country, P_{Ht}. In this derivation, the home consumer's consumption basket, C_{Ht}, consists of goods produced by the home firms z where we integrate from 0 to a because there are a home firms:

min $p_t(z)c_t(z)$ subject to $C_{Ht} = 1$ where $C_{Ht} = [(\frac{1}{a})^{\frac{1}{\theta}} \int_0^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}}$

$L = p_t(z)c_t(z) - P_{Ht}[[(\frac{1}{a})^{\frac{1}{\theta}} \int_o^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}} - 1]$

$\frac{\partial L}{\partial c_t(z)} = p_t(z) - P_{Ht}\frac{\theta}{\theta-1}[(\frac{1}{a})^{\frac{1}{\theta}} \int_o^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}-1}(\frac{1}{a})^{\frac{1}{\theta}}\frac{\theta-1}{\theta}c_t(z)^{\frac{\theta-1}{\theta}-1} = 0$

$p_t(z) = P_{Ht}[(\frac{1}{a})^{\frac{1}{\theta}} \int_o^a c_t(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{1}{\theta-1}}(\frac{1}{a})^{\frac{1}{\theta}}c_t(z)^{-\frac{1}{\theta}}$

$c_t(z)^{-\frac{1}{\theta}} = \frac{p_t(z)}{P_{Ht}}a^{\frac{1}{\theta}}$

$c_t(z) = \frac{1}{a}(\frac{p_t(z)}{P_{Ht}})^{-\theta}$ [2]

Substitute this expression into $C_{Ht} = 1$:

$[(\frac{1}{a})^{\frac{1}{\theta}} \int_o^a (\frac{P_{Ht}}{p_t(z)})^{\theta-1}(\frac{1}{a})^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}} = 1$

$[(\frac{1}{a})^{\frac{1}{\theta}}(\frac{1}{a})^{\frac{\theta-1}{\theta}} \int_o^a (\frac{P_{Ht}}{p_t(z)})^{\theta-1} dz]^{\frac{\theta}{\theta-1}} = 1$

$P_{Ht}[\frac{1}{a} \int_o^a (\frac{1}{p_t(z)})^{\theta-1} dz]^{\frac{\theta}{\theta-1}} = 1$

$[\frac{1}{a} \int_o^a p_t(z)^{1-\theta} dz]^{-\frac{\theta}{\theta-1}} = P_{Ht}$

$[\frac{1}{a} \int_o^a p_t(z)^{1-\theta} dz]^{-\frac{1}{\theta-1}} = P_{Ht}$

$[\frac{1}{a} \int_o^a p_t(z)^{1-\theta} dz]^{\frac{1}{1-\theta}} = P_{Ht}$, which is the price index of goods produced by home firms (denoted by z) in the home country.

We can then write the demand for home firm z output by the representative household in the home country based on the above as:

$c_t(z) = \frac{1}{a}(\frac{p_t(z)}{P_{Ht}})^{-\theta} C_{Ht} = \frac{1}{a}(\frac{p_t(z)}{P_{Ht}})^{-\theta}(\frac{P_{Ht}}{P_t})^{-\omega} aC_t = (\frac{p_t(z)}{P_{Ht}})^{-\theta}(\frac{P_{Ht}}{P_t})^{-\omega} C_t$[3].

Since there are a home households, the demand for home firm z output by all households in the home country is: $(\frac{p_t(z)}{P_{Ht}})^{-\theta}(\frac{P_{Ht}}{P_t})^{-\omega} aC_t$.

[2]Note that this expression should be completely written as $c_t(z) = \frac{1}{a}(\frac{P_{Ht}}{p_t(z)})^{\theta} C_{Ht}$ but we drop C_{Ht} because we imposed $C_{Ht} = 1$.

[3]Note that in this expression we should write $(C_t + G_t)$ to reflect the total demand made by the home country that comes from home consumers as well as home government. However, for the purpose of this derivation, we can omit G_t.

The demand for home firm z output by all households and government in the home country is: $(\frac{p_t(z)}{P_{Ht}})^{-\theta}(\frac{P_{Ht}}{P_t})^{-\omega}(aC_t + aG_t)$ assuming that the government spends G_t per capita. Notice: $a(C_t + G_t)$ is Y_t^d, i.e, demand for consumption basket in the home country. Note that in contrast to Ghironi, Lee, and Rebucci (2015) (GLR), we do not have Y_t^W. Note: The total per capita demand for consumption basket in the home country is $Y_t^d = C_t + G_t$

The price index of goods produced by foreign firms in the home country can be derived by following the same steps. In this derivation, the home consumer's consumption basket, C_{Ft}, consists of goods produced by the foreign firms z^* where we integrate from a to $1-a$ because there are $1-a$ foreign firms:
$[\frac{1}{1-a}\int_a^1 p_t(z^*)^{1-\theta}\,dz^*]^{\frac{1}{1-\theta}} = P_{Ft}$ using consumption of goods produced by foreign firms in the home country, $C_{Ft} = [(\frac{1}{1-a})^{\frac{1}{\theta}}\int_a^1 c_t(z^*)^{\frac{\theta-1}{\theta}}\,dz^*]^{\frac{\theta}{\theta-1}}$

The derivation of the price index of goods produced in the foreign country (consisting of a price index of goods produced by home firms in the foreign country, P^*_{Ht}, and a price index of goods produced by foreign firms in the foreign country, P^*_{Ft}), P^*_t, yields:
$P^*_t = [aP^{*\,1-\omega}_{Ht} + (1-a)P^{*\,1-\omega}_{Ft}]^{\frac{1}{1-\omega}}$

Note that the expressions for P_{Ht}, P_{Ft}, P^*_{Ht} and P^*_{Ft} (and, hence, P_t and P^*_t) are identical to GLR. However, since purchasing power parity does not hold in our model, we have to take the real exchange rate, Q_t, into account.
$Q_t \equiv \frac{\varepsilon_t P^*_t}{P_t}$ where ε_t is the nominal exchange rate, and $\varepsilon_t P^*_t = [a(\varepsilon_t P^*_{Ht})^{1-\omega} + (1-a)(\varepsilon_t P^*_{Ft})^{1-\omega}]^{\frac{1}{1-\omega}}$.
Then: $Q_t = [\frac{a(\varepsilon_t P^*_{Ht})^{1-\omega} + (1-a)(\varepsilon_t P^*_{Ft})^{1-\omega}}{aP^{1-\omega}_{Ht} + (1-a)P^{1-\omega}_{Ft}}]^{\frac{1}{1-\omega}}$

1.2 Household optimization

Start with the home household budget constraint in nominal terms in home currency:

$$(V_t + D_t + \varepsilon_t D_t)x_t + (\varepsilon_t V^*_t + D^*_t + \varepsilon_t D^*_t)x^*_t + W_t L_t = V_t x_{t+1} + \varepsilon_t V^*_t x^*_{t+1} + P_t C_t + P_t G_t,$$

where x_t denotes shares of the home firm, x^*_t denotes shares of the foreign firm, V_t is the price of the home firm's shares, V^*_t is the price of the foreign firm's shares, D_t is the dividend

With respect to x_{t+1} :

[redacted]

where the de nition of R_{t+1} di ers from GLR due to the home rm's dividends coming from two di erent countries.

With respect to x_{t+1} :

$$\frac{@}{@x_{t+1}} = {}_t(\; v_t) + \; E_t f \; {}_{t+1}(v_{t+1} + d_{t+1} + d_{t+1}) g = 0$$

subject to:

$Y_t^s(z) = Y_t^d(z)$, which says that output supplied by the home rm in the home country has to equal this rm's output demanded in the home country,

and

$Y_t^s\ (z) = Y_t^d\ (z)$, which says that output supplied by the home rm in the foreign country has to equal this rm's output demanded in the foreign country.

To derive the optimal demand for labor by home rm, z, in the home country, we use $Y_t^s(z) = Y_t^d(z)$. $Y_t^s(z)$ comes from the production function, i.e.,$Y_t^s(z) = Z_t L_t(z)$. $Y_t^d(z)$ comes from the demand for home rm's z good that was derived in Section 1.1: $Y_t^d(z) = \left(\frac{p_t(z)}{P_{Ht}}\right) \quad \left(\frac{P_{Ht}}{P_t}\right)^{!} (aC_t + aG_t)$ (which is $Y_t^d(z) = \left(\frac{p_t(z)}{P_{Ht}}\right) \quad \left(\frac{P_{Ht}}{P_t}\right)^{!} Y_t^d$ because $(aC_t + aG_t)$ represents the demand by all home fi39

aC .83]TJ/F9 7.9701 Tf 15.378 -1.794 Td [(t)]TJ/F18 11.69 -1.339 Td [(H)-75(t)]TJ ET q 1 0 0 1 359.842 50

zz)Td05 cm []0 d 0 J 0.478 w 0 0 0468.473 0 l S Q BT /F35 7.9701 Tf 320.268 496.794 Td [(

subject to:

$Y_t^s($

$p_t(z) = \frac{}{1}\frac{W_t}{Z_t\,Z_t^{\;1}}$, which is the price charged by the home rm in the foreign country.

For the foreign rm, z , the problem becomes:

$$\text{Max } p_t(z\ )Z_t^{1}\quad Z_t\ (\tfrac{p_t(z\ )}{P_{Ft}})\quad(\tfrac{P_{Ft}}{P_t})\ !\ \tfrac{a(C_t+G_t)}{Z_t^{1}\quad Z_t} + "_t p_t(z\ )Z_t\,(\tfrac{p_t(z\ )}{P_{Ft}})\quad(\tfrac{P_{Ft}}{P_t})\ !\ \tfrac{(1\quad a)(C_t+G_t)}{Z_t}$$
$$W_t(\tfrac{p_t(z\ )}{P_{Ft}})\quad(\tfrac{P_{Ft}}{P_t})\ !\ \tfrac{a(C_t+G_t)}{Z_t^{1}\quad Z_t}\quad "_t W_t\,(\tfrac{p_t(z\ )}{P_{Ft}})\quad(\tfrac{P_{Ft}}{P_t})\ !\ \tfrac{(1\quad a)(C_t+G_t)}{Z_t}$$

Take the derivative with respect to $p_t(z\ )$:

$(1\qquad) = \frac{W_t}{Z_t^{1}\quad Z_t\quad p_t(z\ )}$

$p_t(z\ ) = \frac{}{1}\frac{W_t}{Z_t^{1}\quad Z_t}$, which is the price charged by the foreign rm in the home country.

Take the derivative with respect to $p_t(z\ )$:

$(1\qquad) = \frac{W_t}{Z_t\,p_t(z\ )}$

$p_t(z\ ) = \frac{}{1}\frac{W_t}{Z_t}$, which is the price charged by the foreign rm in the foreign country.

In equilibrium, $p_t(z) = P_{Ht}$, which says that price charged by home rm z in home country equals the price index for goods produced by home rms. Similarly, $p_t(z) = P_{Ht}$ for price charged by home rms in the foreign country, $p_t(z\ ) = P_{Ft}$ for price charged by foreign rms in the home country, and $p_t(z\ ) = P_{Ft}$ for price charged by foreign rms in the foreign country.

Therefore:

$P_{Ht} = \frac{}{1}\frac{W_t}{Z_t}$ for price index of goods produced by home rms in the home country,

$P_{Ht} = \frac{}{1}\frac{W_t}{Z_t\,Z_t^{\;1}}$ for price index of goods produced by home rms in the foreign country,

$P_{Ft} = \frac{}{1}\frac{W_t}{Z_t^{1}\quad Z_t}$ for price index of goods produced by foreign rms in the home country, and

$P_{Ft} = \frac{}{1}\frac{W_t}{Z_t}$ for price index of goods produced by foreign rms in the foreign country.

Then, we can write expressions for relative prices:

$RP_t = \frac{p_t(z)}{P_t} = \frac{P_{Ht}}{P_t} = \frac{}{1}\frac{w_t}{Z_t}$ for price charged by a home rm in the home country relative to the home country's price level in units of the home country consumption,

$RP_t = \frac{p_t(z)}{P_t} = \frac{P_{Ht}}{P_t} = \frac{}{1}\frac{w_t}{Z_t\,Z_t^{\;1}}$ for price charged by a home rm in the foreign country relative to the foreign country's

$RP_t = \frac{p_t(z)}{P_t} = \frac{P_{Ft}}{P_t} = \frac{}{1} \frac{w_t}{Z_t^1 \quad Z_t}$ harged by a foreign rm ntry

relative to the home country's p nits of the home countr

$RP_t = \frac{p_t(z)}{P_t} = \frac{P_{Ft}}{P_t} = \frac{}{1} \frac{w_t}{Z_t}$ f ged by a foreign rm i

relative to the foreign country's units of the foreign cou mp

Note that the small case letter,w denote real wage as op he la

letter W that denotes nominal w

The optimal labor demands can with relative prices as:

Optimal demand for labor by a h country be

$L_t(z) = RP_t^{!} \frac{a(C_t + G_t)}{Z_t}$ z) = RP

$$(1 \quad a)L_t\ (z\ ) = (1 \quad a)RP_t^{\ !}\ \frac{a(C_t + G_t)}{Z_t^1 \quad Z_t}$$

Per capita labor demand by all foreign rms in home country is:

$$\frac{1 \quad a}{a}L_t\ (z\ ) = \frac{1 \quad a}{a}RP_t^{\ !}\ \frac{a(C_t + G_t)}{Z_t^1 \quad Z_t}$$

where we again divide by a because there are a households in the home country.

There are a home rms in the foreign country, so the optimal demand for labor by all home rms in the foreign country is:

$$aL_t(z) = aRP_t^{\ !}\ \frac{(1 \quad a)(C_t + G_t)}{Z_t\ Z_t^{\ 1}}$$

Per capita labor demand by all home rms in foreign country is:

$$\frac{a}{1 \quad a}L_t(z) = \frac{a}{1 \quad a}RP_t^{\ !}\ \frac{(1 \quad a)(C_t + G_t)}{Z_t\ Z_t^{\ 1}}$$

where we divide by $(1 \quad a)$ because there are $(1 \quad a)$ households in the home country.

There are $(1 \quad a)$ foreign rms in the foreign country, so the optimal demand for labor by all foreign rms in the foreign country is:

$$(1 \quad a)L_t(z\ ) = (1 \quad a)RP_t^{\ !}\ \frac{(1 \quad a)(C_t + G_t)}{Z_t}$$

Total per capita labor demand by all foreign rms in the foreign country is:

$$\frac{1 \quad a}{1 \quad a}L_t(z\ ) = \frac{1 \quad a}{1 \quad a}RP_t^{\ !}\ \frac{(1 \quad a)(C_t + G_t)}{Z_t}$$

where we again divide by $(1 \quad a)$ because there are $(1 \quad a)$ households in the home country.

1.4 Net foreign assets (NFA) law of motion

Start with the home household budget constraint in units of the home country's consumption basket from Section 1.2:

$$(v_t + d_t + d_t)x_t + (v_t + d_t + d_t)x_t + w_t L_t = v_t x_{t+1} + v_t x_{t+1} + C_t + G_t$$

Then:

$$(v_t + d_t + d_t)x_t + (v_t + d_t + d_t)x_t + w_t L_t = v_t x_{t+1} + nfa_{t+1} + \tfrac{1-a}{a} v_t x_{t+1} + C_t + G_t$$

where net foreign assets, nfa_{t+1}, is defined as $nfa_{t+1} \equiv v_t x_{t+1} - \frac{1-a}{a} v_t x_{t+1}$, i.e., the value of home holdings of foreign shares minus the value of foreign holdings of home shares adjusted for population sizes of home and foreign countries, i.e., a, and $1-a$, respectively, as in GLR.

We defined return on holding home equity as $R_t \equiv \frac{v_t + d_t + d_t}{v_{t-1}}$ and return on holding foreign equity as $R_t \equiv \frac{v_t + d_t + d_t}{v_{t-1}}$ in Section 1.2, so:

$$v_t x_{t+1} + nfa_{t+1} + \tfrac{1-a}{a} v_t x_{t+1} + C_t + G_t = \frac{(v_t + d_t + d_t)v_{t-1}}{v_{t-1}} x_t + \frac{(v_t + d_t + d_t)v_{t-1}}{v_{t-1}} x_t + w_t L_t$$

$$v_t x_{t+1} + nfa_{t+1} + \tfrac{1-a}{a} v_t x_{t+1} + C_t + G_t = R_t v_{t-1} x_t + R_t v_{t-1} x_t + w_t L_t$$

$$nfa_{t+1} = -v_t x_{t+1} - \tfrac{1-a}{a} v_t x_{t+1} + R_t v_{t-1} x_t + R_t v_{t-1} x_t + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = -v_t (x_{t+1} + \tfrac{1-a}{a} x_{t+1}) + R_t v_{t-1} x_t + R_t v_{t-1} x_t + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = -v_t + R_t v_{t-1} x_t + R_t v_{t-1} x_t + w_t L_t - C_t - G_t$$

where market clearing condition $a x_{t+1} + (1-a) x_{t+1} = a$ was used to obtain $x_{t+1} = 1 - \frac{1-a}{a} x_{t+1}$ as in GLR.

$$nfa_{t+1} = -v_t + R_t v_{t-1} x_t + R_t v_{t-1}(1 - \tfrac{1-a}{a} x_t) + w_t L_t - C_t - G_t \text{ where we used } x_t = 1 - x_t \tfrac{1-a}{a}.$$

$$nfa_{t+1} = -v_t + R_t v_{t-1} x_t + R_t v_{t-1} - R_t v_{t-1} \tfrac{1-a}{a} x_t + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = -v_t + R_t v_{t-1} x_t + v_t + d_t + d_t - R_t v_{t-1} \tfrac{1-a}{a} x_t + w_t L_t - C_t - G_t$$

$$nfa_{t+1} = R_t v_{t-1} x_t - R_t v_{t-1} \tfrac{1-a}{a} x_t + y_t - C_t - G_t$$

where $y_t \equiv d_t + d_t + w_t L_t$, which differs from GLR due to the additional term d_t. Note that we assume that the dividend of the home firm producing in the foreign country, d_t, is a part of the home country GDP, i.e., we assume that firms repatriate profits to their countries of origin for distribution to domestic and foreign shareholders.

$$nfa_{t+1} = R_t v_{t-1} x_t - R_t v_{t-1} x_t + R_t v_{t-1} x_t - R_t v_{t-1} \tfrac{1-a}{a} x_t + y_t - C_t - G_t$$

Define excess return from holding foreign equity $R_t^D = R_t - R_t$ and portfolio holding $_t = v_{t-1} x_t$:

$$nfa_{t+1} = R_t^D \,{}_t + R_t v_{t-1} x_t - R_t v_{t-1} \tfrac{1-a}{a} x_t + y_t - C_t - G_t$$

$$nfa_{t+1} = R_t^D \,{}_t + R_t nfa_t + y_t - C_t - G_t$$

where definition $nfa_t \equiv v_{t-1} x_t - \frac{1-a}{a} v_{t-1} x_t$ was used.

This is identical to GLR except the definitions of R_t and R_t, and hence R_t^D, differ as explained above. This is in units of home consumption.

Similar derivations can be done to obtain the NFA law of motion for the foreign household:

$$nfa_{t+1}^{f} = R_t^{Df} \quad {}_t^{f} + R_t^{f} nfa_t^{f} + y_t^{f} \quad C_t^{f}$$

Derivation of home GDP, y_t, i.e., output produced by home and foreign rms in the home country:

$$y_t = RP_t Z_t L_t + RP_t Z_t^1 \quad Z_t \quad L_t = \frac{}{1}\frac{w_t}{Z_t} Z_t L_t + \frac{}{1}\frac{w_t}{Z_t^1 \quad Z_t} Z_t^1 \quad Z_t \quad L_t = \frac{}{1}(w_t L_t + w_t L_t),$$

which is in units of home country consumption.

Derivation of foreign GDP, y_t, i.e., output produced by home and foreign rms in the foreign country:

$$y_t = RP_t Z_t Z_t^{1} \quad L_t + RP_t Z_t L_t = \frac{}{1}\frac{w_t}{Z_t Z_t^{1}} Z_t Z_t^{1} \quad L_t + \frac{}{1}\frac{w_t}{Z_t} Z_t L_t =$$

$$= \frac{}{1}(w_t L_t + w_t L_t),$$

which is in units of foreign country consumption.

Expression for $\frac{y_t}{y_t}$:

$$\frac{y_t}{y_t} = \frac{RP_t Z_t L_t + RP_t Z_t^1 \quad Z_t \quad L_t}{RP_t Z_t Z_t^{1} \quad L_t + RP_t Z_t L_t} = \frac{\frac{}{1}(w_t L_t + w_t L_t)}{\frac{}{1}(w_t L_t + w_t L_t)} = \frac{w_t (L_t + L_t)}{w_t (L_t + L_t)}$$

Note that we should use the real exchange rate in the relative GDP expression but it cancels because it appears on both sides of the equation $\frac{y_t}{Q_t y_t} = \frac{w_t (L_t + L_t)}{Q_t w_t (L_t + L_t)}$

Next, expressions for w_t, w_t, $(L_t + L_t)$ and $(L_t + L_t)$ are obtained. To get w_t, home labor supply and home labor demand are equated. Home labor supply was derived in Section 1.2 from home household FOCs as $L_t^s = (\frac{C_t^{\frac{1}{}} w_t}{})'$. Home labor demand was derived above from rm FOCs in Section 1.3 as $L_t^d = RP_t \; ! \; \frac{a(C_t + G_t)}{}$

(C[1]

$$\frac{y_t}{y_t} = [\frac{C_t + G_t}{C_t + G_t}]^{\frac{1\ \ !}{'\ +\ !}} [\frac{aZ_t^{!\ \ 1} + (1\ \ a)(Z_t^{1}\ \ Z_t\ \ )^{!\ \ 1}}{a(Z_t\ Z_t^{\ 1}\ \ )^{!\ \ 1} + (1\ \ a)Z_t^{\ !\ \ 1}}]^{\frac{1+\ '}{!\ \ 1}\frac{1\ \ !}{'\ +\ !}} [\frac{C_t + G_t}{C_t + G_t}]^{\frac{1+\ '}{'\ +\ !}} [\frac{aZ_t^{!\ \ 1} + (1\ \ a)(Z_t^{1}\ \ Z_t\ \ )^{!\ \ 1}}{a(Z_t\ Z_t^{\ 1}\ \ )^{!\ \ 1} + (1\ \ a)Z_t^{\ !\ \ 1}}]^{\frac{1+\ '}{'\ +\ !}} =$$

$$= [\frac{C_t + G_t}{C_t + G_t}]^{\frac{(!\ \ 1)+(1+\ \ ')}{'\ +\ !}} + [\frac{aZ_t^{!\ \ 1} + (1\ \ a)(Z_t^{1}\ \ Z_t\ \ )^{!\ \ 1}}{a(Z_t\ Z_t^{\ 1}\ \ )^{!\ \ 1} + (1\ \ a)Z_t^{\ !\ \ 1}}]^{\frac{(1+\ '\ )(1\ \ !\ )+(1+\ \ '\ )(!\ \ 1)}{(!\ \ 1)('\ +\ !\ )}} =$$

$$= \frac{C_t + G_t}{C_t + G_t}$$

1.6 More on real exchange rate, Q_t

From Section 1.1: $Q_t = [\frac{a("_t P_{Ht}\ )^{1\ \ !} + (1\ \ a)("_t P_{Ft}\ )^{1\ \ !}}{aP_{Ht}^{1\ \ !} + (1\ \ a)P_{Ft}^{1\ \ !}}]^{\frac{1}{1\ \ !}}$.

$$Q_t^{1\ \ !} = \frac{a("_t P_{Ht}\ )^{1\ \ !} + (1\ \ a)("_t P_{Ft}\ )^{1\ \ !}}{aP_{Ht}^{1\ \ !} + (1\ \ a)P_{Ft}^{1\ \ !}}$$

Use expressions for price indices:

$$Q_t^{1\ \ !} = \frac{a("_t \frac{}{1} \frac{W_t}{Z_t\ Z_t^{\ 1}})^{1\ \ !} + (1\ \ a)("_t \frac{}{1} \frac{W_t}{Z_t})^{1\ \ !}}{a(\frac{}{1} \frac{W_t}{Z_t})^{1\ \ !} + (1\ \ a)(\frac{}{1} \frac{W_t}{Z_t^{1}\ \ Z_t})^{1\ \ !}} = (\frac{"_t W_t}{W_t})^{1\ \ !} \frac{a(Z_t\ Z_t^{\ 1}\ \ )^{!\ \ 1} + (1\ \ a)Z_t^{\ !\ \ 1}}{aZ_t^{!\ \ 1} + (1\ \ a)(Z_t^{1}\ \ Z_t\ \ )^{!\ \ 1}} =$$

$$= (\frac{\frac{"_t W_t}{P_t} P_t}{\frac{W_t}{P_t} P_t})^{1\ \ !} \frac{a(Z_t\ Z_t^{\ 1}\ \ )^{!\ \ 1} + (1\ \ a)Z_t^{\ !\ \ 1}}{}$$

$)^{1\ \ !}\ a(Z\ {}^{t} Z_t^{\ 1}\ \ )^{!\ \ 1} + (1\ \ a)Z$

$aZ_t^{!\ \ 1} + (1\ \ a)(Z_t^{1}$

$$\left(\frac{a}{1\ \ a}\right)^{\frac{!\ +\ '\ \ \ \ (!\ \ \ \ 1)}{!\ \ \ \ 1}}\left[\ ^{Z}\right.$$

$_{b}bC_{db}$ **bC** $-E\ b^{bC}$ 8000

$+ E\ b^{b}Q_t$

$E_t\ b\hat{C}_t$

Similarly, foreign GDP y_t, i.e., output produced by home and foreign rms in the foreign country, equals $y_t = \frac{}{1}(w_t L_t + w_t L_t)$ in units of foreign country consumption. Labor income, therefore, equals $\frac{1}{}y_t$. In units of home country consumption, this is $\frac{1}{}y_t Q_t$. The pro t of foreign rms, i.e., the pro t generated by foreign rms in home and foreign countries, $d_t + d_t$, in units of home country consumption is then $\frac{1}{}y_t Q_t$, which again shows that the share of rm pro ts, i.e., the dividend income, in the foreign GDP is a constant proportion $\frac{1}{}$.

2 Model Solution

There are four variables that will determine the model solution: C_t^D, Q_t, y_t^D, and nfa_{t+1}.

2.1 Log-linearize Euler equations for consumption

Section 1.2 shows FOC wrt x_{t+1} combined with FOC wrt C_t, which gives the Euler equation:

$C_t^{\frac{1}{}} = E_t f C_{t+1}^{\frac{1}{}} R_{t+1} g$

Log-linearize this equation:

$\frac{1}{}\hat{C}_t = \frac{1}{}E_t\hat{C}_{t+1} + E_t\hat{R}_{t+1}$

Similarly, the Euler equation for the foreign country is:

$C_t^{\frac{1}{}} = E_t f C_{t+1}^{\frac{1}{}} R_{t+1}^f g = E_t f C_{t+1}^{\frac{1}{}} R_{t+1} \frac{Q_t}{Q_{t-1}} g$

where we used $R_t^f = \frac{v_t^f + d_t^f + d_t^f}{v_{t-1}^f}$ de ned in Section 1.4 and $R_{t+1} = \frac{Q_{t+1}}{Q_t}R_{t+1}^f$ wh17.219 7. Td [1 Tf45r1

$$E_t(\widehat{C}^D_{t+1} \quad \widehat{C}^D_t) = E_t(\widehat{Q}_{t+1} \quad \widehat{Q}_t)$$

2.2 Log-linearize expression from Section 1.6 and nd elasticities of $\widehat{C}^D_t$

This derivation nds elasticities of $\widehat{C}^D_t$:

$$\frac{C_t + G_t}{C_t + G_t}\left(\frac{C_t}{C_t}\right)^{\frac{'}{}} = \left[\frac{aZ_t^{!\ 1} + (1\ \ a)(Z_t^{1}\ \ Z_t\ \ )^{!\ \ 1}}{a(Z_t\ Z_t^{\ 1}\ \ )^{!\ \ 1} + (1\ \ a)Z_t^{\ !\ \ 1}}\right]^{\frac{1+'}{!\ \ 1}}$$

$$\log(C_t + G_t)\ \ \log(C_t + G_t) + \frac{'}{}(\log C_t\ \ \log C_t) = \frac{1+'}{!\ \ 1}[\log(aZ_t^{!\ \ 1} + (1\ \ a)(Z_t^1\ \ Z_t\ \ )^{!\ \ 1})\ \ \log(a(Z_t\ Z_t^{\ 1}\ \ )^{!\ \ 1} + (1\ \ a)Z_t^{\ !\ \ 1})]$$

$$\frac{dC_t + dG_t}{C + G}\ \ \frac{dC_t + dG_t}{C + G} + \frac{'}{}\left(\frac{dC_t}{C}\ \ \frac{dC_t}{C}\right) = \frac{1+'}{!\ \ 1}[a(!\ \ 1)dZ_t + (1\ \ a)(!\ \ 1)((1\ \ )dZ_t + dZ_t)\ \ [a(!\ \ 1)(dZ_t + (1\ \ )dZ_t) + (1\ \ a)(!\ \ 1)dZ_t]]$$

Use $Z = Z$, which is true in the symmetric steady state. Normalize $Z = Z$ to 1.

$$\frac{dC_t\frac{C}{C} + dG_t\frac{G}{G}}{C + G}\ \ \frac{dC_t\frac{C}{C} + dG_t\frac{G}{G}}{C + G} + \frac{'}{}\widehat{C}^D_t = \frac{1+'}{!\ \ 1}[a(!\ \ 1)\widehat{Z}_t + (1\ \ a)(!\ \ 1)((1\ \ )\widehat{Z}_t + \widehat{Z}_t)\ \ [a(!\ \ 1)(\widehat{Z}_t + (1\ \ )\widehat{Z}_t) + (1\ \ a)(!\ \ 1)\widehat{Z}_t]$$

$$\frac{C}{C+G}(\widehat{C}_t\ \ \widehat{C}_t) + \frac{G}{C+G}(\widehat{G}_t\ \ \widehat{G}_t) + \frac{'}{}\widehat{C}^D_t = \frac{1+'}{!\ \ 1}[a(!\ \ 1)\widehat{Z}_t + (1\ \ a)(!\ \ 1)(1\ \ )\widehat{Z}_t + (1\ \ a)(!\ \ 1)\widehat{Z}_t\ \ a(!\ \ 1)\widehat{Z}_t\ \ a(!\ \ 1)(1\ \ )\widehat{Z}_t\ \ (1\ \ a)(!\ \ 1)\widehat{Z}_t]$$

Use $y = C + G$. Since $y = 1$, $C + G = 1$ and $C = 1\ \ G$. Then,

$$(1\ \ G)(\widehat{C}_t\ \ \widehat{C}_t) + G(\widehat{G}_t\ \ \widehat{G}_t) + \frac{'}{}\widehat{C}^D_t = \frac{1+'}{!\ \ 1}[a(!\ \ 1)(1\ \ )\widehat{Z}_t + (1\ \ a)(!\ \ 1)(1\ \ )\widehat{Z}_t\ \ (1\ \ a)(!\ \ 1)(1\ \ )\widehat{Z}_t\ \ a(!\ \ 1)(1\ \ )\widehat{Z}_t$$

$$(1\ \ G)\widehat{C}^D_t + G\widehat{G}^D_t + \frac{'}{}\widehat{C}^D_t = \frac{1+'}{!\ \ 1}(!\ \ 1)(1\ \ )(\widehat{Z}_t\ \ \widehat{Z}_t)$$

$$(1\ \ G)\widehat{C}^D_t + G\widehat{G}^D_t + \frac{'}{}\widehat{C}^D_t = (1 + ')(1\ \ )\widehat{Z}^D_t$$

$$(1\ \ G + \frac{'}{})\widehat{C}^D_t + = (1 + ')(1\ \ )\widehat{Z}^D_t\ \ G\widehat{G}^D_t$$

$$\widehat{C}^D_t = \frac{(1+')(1\ \ )}{1\ \ G + \frac{'}{}}\widehat{Z}^D_t\ \ \frac{G}{1\ \ G + \frac{'}{}}\widehat{G}^D_t$$

$$\widehat{C}^D_t = {}_{C^D Z^D}\widehat{Z}^D_t + {}_{C^D G^D}\widehat{G}^D_t$$

If $G = 0$ (i.e., no scal shocks), $\widehat{C}^D_t = \frac{(1+')(1\ \ )}{1+\frac{'}{}}\widehat{Z}^D_t$

If $G = 0$ and $' = 0$ (i.e., inelastic labor) $\widehat{C}^D_t = (1\ \ )\widehat{Z}^D_t$

If $G = 0$, $' = 0$, and $= 1$, $\widehat{C}^D_t = 0$.

If $G = 0$, $' = 0$, and $= 0$, $\widehat{C}^D_t = \widehat{Z}^D_t$.

If $G = 0$ and $= 1$, $\widehat{C}^D_t = 0$ regardless of $'$.

If $G \neq 0$ and $' = 0$, $\widehat{C}^D_t = \frac{(1\ \ )}{1\ \ G}\widehat{Z}^D_t\ \ \frac{G}{1\ \ G}\widehat{G}^D_t$

If $G \neq 0$, $' = 0$ and $= 1$, $\widehat{C}^D_t = \ \ \frac{G}{1\ \ G}\widehat{G}^D_t$. If $= 0$, $\widehat{C}^D_t = \frac{1}{1\ \ G}\widehat{Z}^D_t\ \ \frac{G}{1\ \ G}\widehat{G}^D_t$.

2.3 Find elasticities of $\hat{Q}_t$

This derivation uses the log-linearized Euler equations from Section 2.1 and $\hat{C}^D_t$ from Section 2.2 to find elasticities of $\hat{Q}_t$:

$E_t(\hat{C}^D_{t+1} \quad \hat{C}^D_t) = E_t(\hat{Q}_{t+1} \quad \hat{Q}_t)$ from Section 2.1.

Combine with $\hat{C}^D_t = {}_{C^D Z^D}\hat{Z}^D_t + {}_{C^D G^D}\hat{G}^D_t$ from 2.2.

$E_t(\hat{Q}_{t+1} \quad \hat{Q}_t) = E_t[\ {}_{C^D Z^D}(\hat{Z}^D_{t+1} \quad \hat{Z}^D_t) + {}_{C^D G^D}(\hat{G}$

Log-linearized: $\hat{L}_t^{total;D} = \hat{y}_t^D \quad \hat{w}_t^D$.

2.6 Log-linearize NFA LOM

This derivation uses the NFA LOM from Section 1.4 to nd the solution for $\widehat{nfa}_{t+1}$:

$nfa_{t+1} = R_t^D \quad _t + R_t nfa_t + (1 \quad a)[(y_t \quad Q_t y_t^f) \quad (C_t \quad Q_t C_t^f) \quad (G_t \quad Q_t G_t^f)]$

$dnfa_{t+1} = dR_t^D \quad + R^D d \quad _t + dR_t nfa + Rdnfa_t + (1 \quad a)[dy_t \quad (dQ_t y^f + Qdy_t^f) \quad (dC_t$
$(dQ_t C^f + QdC_t^f)) \quad (dG_t \quad (dQ_t G^f + QdG_t^f)]$

Use $R^D = 0$ and $nfa = 0$:

$dnfa_{t+1} = dR_t^D \quad + Rdnfa_t + (1 \quad a)[dy_t \quad (dQ_t y^f + Qdy_t^f) \quad (dC_t \quad (dQ_t C^f + QdC_t^f))$
$(dG_t \quad (dQ_t G^f + QdG_t^f)]$

Use $Q = 1$ because it holds in the symmetric steady state, and net foreign assets equal 0:

$dnfa_{t+1} = dR_t^D \quad + Rdnfa_t + (1 \quad a)[(dy_t \quad (dQ_t y^f + dy_t^f) \quad (dC_t \quad (dQ_t C^f + dC_t^f))$
$(dG_t \quad (dQ_t G^f + dG_t^f)]$

Notice that we are subtracting dy_t and dy_t^f that are in units of home consumption and foreign consumption, respectively. We can subtract these terms because we already accounted for the di erent units by including the real exchange rate. This works because in the symmetric steady state, the real exchange rate terms drop out ($Q = 1$). This is used later on in other derivations, for example, the derivation of the di erential in equity values, $\hat{p}_t^D$.

$dnfa_{t+1} = dR_t^D \quad + Rdnfa_t + (1 \quad a)[(dy_t^D \quad dQ_t y^f) \quad (dC_t^D \quad dQ_t C^f) \quad (dG_t^D \quad dQ_t G^f)]$

Divide by C. Use $C = 1 \quad G$, which comes from $y = C + G$ combined with $y = 1$:

$\frac{dnfa_{t+1}}{C} = \frac{dR_t^D}{1 \quad G} + \frac{Rdnfa_t}{C} + (1 \quad a)[(\frac{dy_t^D}{1 \quad G} \quad \frac{dQ_t y^f}{1 \quad G}) \quad (\frac{dC_t^D}{C} \quad \frac{dQ_t C^f}{C}) \quad (\frac{dG_t^D}{1 \quad G} \quad \frac{dQ_t G^f}{1 \quad G})]$

$\widehat{nfa}_{t+1} = \frac{dR_t^D}{1 \quad G} R$

$$\widehat{nfa}_{t+1} = \frac{\quad}{(1 \quad G)} \hat{R}^D_t + \frac{1}{\quad}\widehat{nfa}_t + \frac{1 \quad a}{1 \quad G}\hat{y}^D_t \quad (1 \quad a)\hat{C}^D_t \quad \frac{(1 \quad a)G}{1 \quad G}\hat{G}^D_t + (1 \quad a)\left[\frac{\hat{Q}_t}{1 \quad G} + \frac{\hat{Q}_t (1 \quad G^f)}{1 \quad G} + \frac{\hat{Q}_t G^f}{1 \quad G} \right]$$

$$\widehat{nfa}_{t+1} = \frac{\quad}{(1 \quad G)} \hat{R}^D_t$$

$$\frac{dv_t}{v} = \frac{dE_t v_{t+1}}{v} + \frac{dE_t d_{t+1}}{v} + \frac{dE_t d_{t+1}}{v}$$

$$\hat{v}_t = E_t \hat{v}_{t+1} + E_t \hat{d}_{t+1} \frac{d}{v} + E_t \hat{d}_{t+1} \frac{d}{v}$$

From Section 1.7, the following holds: $d_t + d_t = \frac{1}{}y_t$. Due to the assumption $y_t = 1$, it is possible to write: $d_t + d_t = \frac{1}{}$. In steady state, the Euler equation for home shares becomes $v = v + d + d$, which becomes $v = v + \frac{1}{}$ which can be written as $v(1 \quad) = -$

Next, we obtain an expression for $\hat{d}^D_{t+1}$. Here, we take advantage of the useful properties from Section 1.7. Since $\bar{d}_t = d_t + d_t = \frac{1}{}y_t$ and $\bar{d}_t = d_t + d_t = \frac{1}{}y_t Q_t$ in units of home country consumption, it is possible to write $\frac{\bar{d}_t}{\bar{d}_t} = \frac{d_t + d_t}{d_t + d_t} = \frac{\frac{1}{}y_t}{\frac{1}{}y_t Q_t}$, which means $\frac{\bar{d}_t}{\bar{d}_t} = \frac{y_t}{y_t Q_t}$.

Roll it forward by one period: $\frac{\bar{d}_{t+1}}{\bar{d}_{t+1}} = \frac{y_{t+1}}{y_{t+1} Q_{t+1}}$.

Log-linearizing gives $\hat{d}^D_{t+1} = \hat{y}_{t+1} \quad (\hat{y}_{t+1} + \hat{Q}_{t+1})$.

Substitute into $\hat{v}^D_t$:

$$\hat{v}^D_t = E_t[\ \hat{v}^D_{t+1} + (1 \quad)(\hat{y}_{t+1} \quad (\hat{y}_{t+1} + \hat{Q}_{t+1}))]$$

Notice: This combines $E_t \hat{R}^D_{t+1} = 0$ and $\hat{R}^D_t = \quad [\ \hat{v}^D_t + (1 \quad)(\hat{y}_t \quad (\hat{y}_t + \hat{Q}_t))] + \hat{v}^D_{t\ 1} =$

$$= \quad [\ \hat{v}^D_t + (1 \quad)(\hat{y}^D_t \quad \hat{Q}_t)] + \hat{v}^D_{t\ 1}$$

$$\hat{v}^D_t = E_t[\ \hat{v}^D_{t+1} + (1 \quad)(\hat{y}^D_{t+1} \quad \hat{Q}_{t+1})]$$

$$\hat{v}^D_t = E_t[\ \hat{v}^D_{t+1} + (1 \quad)(\ {}_{y^D Z^D} \hat{Z}^D_{t+1} + {}_{y^D G^D} \hat{G}^D_{t+1} \quad {}_{QZ^D} \hat{Z}^D_{t+1} \quad {}_{QG^D} \hat{G}^D_{t+1}]$$

$$\hat{v}^D_t = E_t[\ \hat{v}^D_{t+1} + (1 \quad)(\ {}_{y^D Z^D} \hat{Z}^D_{t+1} + {}_{y^D G^D} \hat{G}^D_{t+1} \quad \frac{1}{}\ {}_{C^D Z^D} \hat{Z}^D_{t+1} \quad \frac{1}{}\ {}_{C^D G^D} \hat{G}^D_{t+1}]$$

$$\hat{v}^D_t = E_t[\ \hat{v}^D_{t+1} + (1 \quad)((\ {}_{y^D Z^D} \quad \frac{1}{}\ {}_{C^D Z^D})\hat{Z}^D_{t+1} + (\ {}_{y^D G^D} \quad \frac{1}{}\ {}_{C^D G^D})\hat{G}^D_{t+1}]$$

$$\hat{v}^D_t = \ {}_{v^D Z^D} \hat{Z}^D_t + \ {}_{v^D G^D} \hat{G}^D_t$$

$${}_{v^D Z^D} \hat{Z}^D_t + {}_{v^D G^D} \hat{G}^D_t = E_t[\ \hat{v}^D_{t+1} + (1 \quad)((\ {}_{y^D Z^D} \quad \frac{1}{}\ {}_{C^D Z^D})\hat{Z}^D_{t+1} + (\ {}_{y^D G^D} \quad \frac{1}{}\ {}_{C^D G^D})\hat{G}^D_{t+1}]$$

$${}_{v^D Z^D} \hat{Z}^D_t + {}_{v^D G^D} \hat{G}^D_t = \quad (\ {}_{v^D Z^D}\ {}_Z \hat{Z}^D_t + {}_{v^D G^D}\ {}_G \hat{G}^D_t) + (1 \quad)[(\ {}_{y^D Z^D} \quad \frac{1}{}\ {}_{C^D Z^D})\ {}_Z \hat{Z}^D_t + (\ {}_{y^D G^D} \quad \frac{1}{}\ {}_{C^D G^D})\ {}_G \hat{G}^D_t]$$

where we used $\hat{Z}^D_{t+1} = \ {}_Z \hat{Z}^D_t + \hat{}_{Z^D\,t+1}$ and $\hat{G}^D_{t+1} = \ {}_G \hat{G}^D_t + \hat{}_{G^D\,t+1}$

Match the coe cients:

$${}_{v^D Z^D} = \quad {}_{v^D Z^D}\ {}_Z + (1 \quad)(\ {}_{y^D Z^D} \quad \frac{1}{}\ {}_{C^D Z^D})\ {}_Z$$

$$(1 \quad {}_Z)\ {}_{v^D Z^D} = (1 \quad)(\ {}_{y^D Z^D} \quad \frac{1}{}\ {}_{C^D Z^D})\ {}_Z$$

$${}_{v^D Z^D} = \frac{(1 \quad)\ {}_Z(\ {}_{y^D Z^D} \quad \frac{1}{}\ {}_{C^D Z^D})}{1 \quad {}_Z}$$

$${}_{v^D G^D} = \quad {}_{v^D G^D}\ {}_G + (1 \quad)(\ {}_{y^D G^D} \quad \frac{1}{}\ {}_{C^D G^D})\ {}_G$$

$$(1 \quad {}_G)\ {}_{v^D G^D} = (1 \quad)(\ {}_{y^D G^D} \quad \frac{1}{}\ {}_{C^D G^D})\ {}_G$$

$${}_{v^D G^D} = \frac{(1 \quad)\ {}_G(\ {}_{y^D G^D} \quad \frac{1}{}\ {}_{C^D G^D})}{1 \quad {}_G}$$

2.8 Show that excess return $\hat{R}^D_t$ is a linear function of innovations to relative productivity and government spending

From Section 2.7:

$$\hat{R}^D_{t+1} = \quad [\ \hat{v}^D_{t+1} + (1 \qquad)(\hat{y}^D_{t+1} \qquad b$$

If G $\neq$ 0, ' = 0 and = 1: $\hat{R}^D_{t+1} = \frac{(1 \quad)(1 \quad)G}{(1 \quad _Z)(1 \quad G)} \hat{Z}^D_{t+1} \quad \frac{(1 \quad)G}{(1 \quad _G)(1 \quad G)} \hat{G}^D_{t+1}$

2.9 2nd-order approximation of the portfolio part of the model

From household FOCs for :

$C_t^{\ \frac{1}{}} = \ E_t f\, C_{t+1}^{\frac{1}{}} R_{t+1}\, g$, which can be written as: $\frac{C_t}{}^{\frac{1}{}} = E_t f\, C_{t+1}^{\frac{1}{}} R_{t+1}\, g$

$C_t^{\ \frac{1}{}} = \ E_t f\, C_{t+1}^{\frac{1}{}} R_{t+1}\, g$, which can be written as: $\frac{C_t}{}^{\frac{1}{}} = E_t f\, C_{t+1}^{\frac{1}{}} R_{t+1}\, g$

Equating these two expressions gives us $E_t(C_{t+1}^{\frac{1}{}} R_{t+1}) = E_t(C_{t+1}^{\frac{1}{}} R_{t+1})$, which can be written as $E_t(C_{t+1}^{\frac{1}{}} R_{t+1}) \quad E_t(C_{t+1}^{\frac{1}{}} R_{t+1}) = 0$

Take second-order approximation and evaluate it at steady state:

$E_t(\ \frac{1}{}C_{t+1}^{\frac{1}{} \ 1} dC_{t+1} R_{t+1}) + E_t(C_{t+1}^{\frac{1}{}} dR_{t+1}) \quad E_t(\ \frac{1}{}C_{t+1}^{\frac{1}{} \ 1} dC_{t+1} R_{t+1}) \quad E_t(C_{t+1}^{\frac{1}{}} dR_{t+1}) +$

$+ \frac{1}{2}[\ \frac{1}{}(\ \frac{1}{} \quad 1)C_{t+1}^{\frac{1}{} \ 1 \ 1} d^2C_{t+1} R_{t+1} + C_{t+1}^{\frac{1}{}} 0 + 2(\ \frac{1}{})C_{t+1}^{\frac{1}{} \ 1} dC_{t+1} dR_{t+1}]$

$\frac{1}{2}[\ \frac{1}{}(\ \frac{1}{} \quad 1)C_{t+1}^{\frac{1}{} \ 1 \ 1} d^2C_{t+1} R_{t+1} \quad C_{t+1}^{\frac{1}{}} 0 \quad 2(\ \frac{1}{})C_{t+1}^{\frac{1}{} \ 1} dC_{t+1} dR_{t+1}] =$

$= E_t(\ \frac{1}{}C^{\ \frac{1}{} \ 1} dC_{t+1} \frac{1}{}) + E_t(C^{\ \frac{1}{}} dR_{t+1}) \quad E_t(\ \frac{1}{}C^{\ \frac{1}{} \ 1} dC_{t+1} \frac{1}{}) \quad E_t(C^{\ \frac{1}{}} dR_{t+1}) +$

$+ \frac{1}{2}[\ \frac{1}{}(\ \frac{1}{} \quad 1)C^{\ \frac{1}{} \ 1 \ 1} d^2C_{t+1} \frac{1}{} + 2(\ \frac{1}{})C^{\ \frac{1}{} \ 1} dC_{t+1} dR_{t+1}]$

$\frac{1}{2}[\ \frac{1}{}(\ \frac{1}{} \quad 1)C^{\ \frac{1}{} \ 1 \ 1} d^2C_{t+1} \frac{1}{} + 2(\ \frac{1}{})C^{\ \frac{1}{} \ 1} dC_{t+1} dR_{t+1}] =$

$= E_t(C^{\ \frac{1}{}} dR_{t+1}) \quad E_t(C^{\ \frac{1}{}} dR_{t+1}) + \frac{1}{2}[2(\ \frac{1}{})C^{\ \frac{1}{} \ 1} dC_{t+1} dR_{t+1}] \quad \frac{1}{2}[2(\ \frac{1}{})C^{\ \frac{1}{} \ 1} dC_{t+1} dR_{t+1}]$

$= E_t(C^{\ \frac{1}{}} dR_{t+1}) \quad E_t(C^{\ \frac{1}{}} dR_{t+1}) + [\ \frac{1}{}C^{\ \frac{1}{} \ 1} dC_{t+1} dR_{t+1}] \quad [\ \frac{1}{}C^{\ \frac{1}{} \ 1} dC_{t+1} dR_{t+1}]$

Divide by $C^{\ \frac{1}{}}$ and R.

$\hat{R}_{t+1} \quad \hat{R}_{t+1} + (\ \frac{1}{}\hat{C}_{t+1} \hat{R}_{t+1}) \quad (\ \frac{1}{}\hat{C}_{t+1} \hat{R}_{t+1}) = 0$

The same derivation for the foreign country gives:

$\hat{R}^f_{t+1} \quad \hat{R}^f_{t+1} + (\ \frac{1}{}\hat{C}^f_{t+1} \hat{R}^f_{t+1}) \quad (\ \frac{1}{}\hat{C}^f_{t+1} \hat{R}^f_{t+1}) = 0$

Subtract expressions for the home and foreign countries:

$\hat{R}_{t+1} \quad \hat{R}_{t+1} + (\ \frac{1}{}\hat{C}_{t+1} \hat{R}_{t+1}) \quad (\ \frac{1}{}\hat{C}_{t+1} \hat{R}_{t+1}) \quad [\hat{R}^f_{t+1} \quad \hat{R}^f_{t+1} + (\ \frac{1}{}\hat{C}^f_{t+1} \hat{R}^f_{t+1}) \quad (\ \frac{1}{}\hat{C}^f_{t+1} \hat{R}^f_{t+1})] = 0$

From Section 1.4: $R_{t+1} = \frac{Q_{t+1}}{Q_t} R^f_{t+1}$

Log-linearize: $\hat{R}_{t+1} = \hat{Q}_{t+1} \quad \hat{Q}_t + \hat{R}^f_{t+1}$

Same for the foreign:

Log-linearize: $\hat{R}_{t+1} = \hat{Q}_{t+1} \quad \hat{Q}_t + \hat{R}^f_{t+1}$

Using this, simplify: $(\ \frac{1}{\ }\hat{C}_{t+1}\ \hat{R}_{t+1}) \quad (\ \frac{1}{\ }\hat{C}_{t+1}\ \hat{R}_{t+1}) \quad [\ \frac{1}{\ }\hat{C}^f_{t+1}\ \hat{R}^f_{t+1}) \quad (\ \frac{1}{\ }\hat{C}^f_{t+1}\ \hat{R}^f_{t+1})] = 0$

$\hat{C}_{t+1}\ \hat{R}_{t+1} \quad \hat{C}_{t+1}\ \hat{R}_{t+1} \quad [\hat{C}^f_{t+1}\ \hat{R}^f_{t+1} \quad \hat{C}^f_{t+1}\ \hat{R}^f_{t+1}] = 0$

$\hat{C}_{t+1}\ \hat{R}_{t+1} \quad \hat{C}_{t+1}\ \hat{R}_{t+1} \quad [\hat{C}^f_{t+1}\ (\hat{R}_{t+1} \quad \hat{Q}_{t+1} + \hat{Q}_t) \quad \hat{C}^f_{t+1}\ (\hat{R}_{t+1} \quad \hat{Q}_{t+1} + \hat{Q}_t)] = 0$

$\hat{C}_{t+1}\ \hat{R}_{t+1} \quad \hat{C}_{t+1}\ \hat{R}_{t+1} \quad [\hat{C}^f_{t+1}\ \hat{R}_{t+1} \quad \hat{C}^f_{t+1}\ \hat{R}_{t+1}] = 0$

$E_t(\hat{C}^D_{t+1}\ \hat{R}^D_{t+1}) = 0$

This results is the same as in GLR.

However, notice that there is no in either expression:

Substitute expressions for $\hat{C}^D_{t+1}$ from Section 2.2 (i.e., $\hat{C}^D_{t+1} = \frac{(1+\ ')(1\ \ )}{1\ \ G+\ -}\hat{Z}^D_{t+1} \quad \frac{G}{1\ \ G+\ -}\hat{G}^D_{t+1}$)

and $\hat{R}^D_{t+1}$ from Section 2.8 (i.e., $\hat{R}^D_{t+1} = \frac{(1\ \ )\frac{(1+\ ')(1\ \ )[\ (1\ \ G)\ \ 1]}{(1\ \ G+\ -)}}{1\ \ \ z}\hat{Z}^D_{t+1} \quad \frac{(1\ \ )\frac{G(\ '\ +1)}{(1\ \ G+\ -)}}{1\ \ \ G}\hat{G}^D_{t+1}$

Hence,

$$\hat{y}_t = a\left[\hat{RP}^d_t + \hat{Z}_t + \hat{L}_t\right] + (1-a)\left[\hat{RP}^{d*}_t + (1- \ )\hat{Z}_t + \ \hat{Z}^*_t + \hat{L}^*_t\right]; \qquad (4)$$

$$\hat{y}^*_t = a\left[\hat{RP}^d_t + \ \hat{Z}_t + (1- \ )\hat{Z}^*_t + \hat{L}^*_t\right] + (1-a)\left[\hat{RP}^{d*}_t + \hat{Z}^*_t + \hat{L}^*_t\right]: \qquad (5)$$

Next, take a population-weighted average of equations (4) and (5), and de ne $\hat{y}^W_t$ as:

$$\hat{y}^W_t \quad a\hat{y}_t + (1-a)\hat{y}^*_t =$$

n)Tf 18.18 -1.793 Td [( )]TJ/F24 11.9552 Tf 11.955 0 Td [( )]TJ/F18 11.9552 Tf 6.722 0 Td

$$= a$$

that this is the same system of equations as in GLR. It follows that the change in production structure and demand-fulfillment from the one in GLR to the one we are studying in this paper matters for how world production is allocated between the two countries but not for the overall amount of world production.

References

Ghironi, F., Lee, J., & Rebucci, A. (2015). The valuation channel of external adjustnfences